From the Classroom to the Lab and Back

Instructional Strategies to Improve Children's Early Math Skills

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Overview

The overarching goal of *From the Classroom to the Lab and Back* is to integrate deep mathematical thinking into early childhood classrooms. Our approach to accomplishing this goal is to develop a coherent system of instruction that begins in pre-K and enables children of all social, ethnic, and linguistic backgrounds to attain high levels of academic achievement in elementary school and beyond. At the core of this system are instructional strategies and statistically reliable assessments that integrate research and practice and provide information to preschool teachers that is highly relevant to individual, group, and whole class instruction. Our approach is grounded in research demonstrating that early mathematics skills predict later achievement in mathematics (Duncan et al., 2007; Gunderson, Ramirez, Beilock & Levine, 2012; Jordan, Kaplan, Ramineni & Locuniak, 2009; Jordan, Glutting & Ramineni, 2010; Lee & Burkam 2002), but that children show marked individual differences in their mathematical knowledge at the start of school and these differences are often associated with children's socioeconomic status (SES) (Jordan, Kaplan, Oláh, & Locuniak, 2006; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Lee & Burkam, 2002; Saxe, Guberman, & Gearhart, 1987; Starkey, Klein, & Wakeley, 2004).

Recent research suggests that math instruction and math-related interactions between adults and children can substantially enhance the development of children's mathematical thinking (Clements & Sarama 2008; Griffin, Case & Siegler, 1994; Starkey, Klein & Wakeley, 2004; Gunderson & Levine 2011; Klibanoff, Levine, Huttenlocher, Vasilyeva & Hedges, 2006; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010). Further, we know that children from economically disadvantaged households seem to gain the most from high quality early childhood programming (ECE) (Barnett, 2008; Burger, 2010). However, we know that math instruction, let alone high-quality mathematics interaction, is rare in early childhood classrooms (Graham, Nash & Paul, 1997; Winton & Buysse, 2005).

We seek to change this state of affairs by clarifying instructional goals for early mathematics development in preschool settings. We will equip teachers with a system that will provide frequent, detailed information about individual children's progress in acquiring critical math concepts and skills and offer strategies to further the development of these skills. One of the ways in which the project gauges student learning is through a mathematics assessment system for 3 to 5 year old preschool children being developed in a related project, *Getting on Track Early for School Success*. Once the assessment system and our instructional strategies are fully developed, we will be able to support teachers in gaining an understanding of a) the critical mathematics content for young children, b) how learning within different content areas progresses from ages 3 to 5, and c) how to use specific instructional strategies and math lessons to advance the acquisition of fundamental math skills and concepts in their students.

We are in an excellent position at the University of Chicago to accomplish this work, which requires ongoing collaboration between researchers and practitioners. We have an established team of early learning scholars from the Committee on Education, developers of mathematics early learning tools, from Center for Elementary Mathematics and Science Education, and practitioners from the University's charter schools and other Chicago Public schools who bring considerable knowledge, experience, and skill to develop our assessment and instruction system.

Between 2011 and 2013, with funding from the McCormick Foundation and the Chicago Mercantile Exchange Foundation, we:

- Collaborated with teachers to explore ways to implement research findings in the classroom and create instructional activities
- Developed and tested a specific learning activity designed to target the numerical skills of cardinality and the successor function and piloted the activity with approximately 300 preschool children
- Developed a learning activity to promote understanding the attributes of a particular shape and piloted the activity with approximately 180 preschool children
- Conducted a study to investigate the role of language in children's misconceptions regarding early geometric concepts
- Completed two pilot studies investigating the relative benefits of action vs. gesture as a way to improve mental rotation ability
- Developed exemplar lessons for preschool teachers

Between 2013 and 2015, *From the Classroom to the Lab and Back* will move forward in several ways:

- We will conduct new lab studies that test instructional strategies in new content areas
- We will identify successful strategies, based on research and piloting outcomes, and incorporate them into prekindergarten and kindergarten math lessons
- We will begin to merge our math strategies and our assessment work by linking the strategies and lessons to specific skills, thereby providing teachers with activities to advance the development of that skill

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Writing and testing research-based exemplar lessons for preschool teachers

During the first two years of this project, we conducted research and developed and piloted instructional strategies that help preschool children learn the numerical, geometric and spatial skills that are critical to future achievement, particularly in the STEM (science, technology, engineering and mathematics) disciplines. We developed 4 pairs of exemplar lessons (a total of 8 lessons) for preschool teachers, focused on geometry and spatial reasoning skills. Two of the lessons involved teaching children about object transformation (mental rotation), and two focused on helping children notice and describe parts of shapes, and use that to refine their understanding of the shape category "triangle," which we know to be very difficult for children at this age.

For each of these four topics, two lesson versions were created. One included a general study protocol that incorporated many different kinds of support to help teachers use gesture, analogy, comparison, and other teaching strategies that lab research suggested help children learn. The second version of each lesson was identical in content, but did not include such supports. Our hypothesis was that these kinds of supports would help teachers incorporate evidence-supported practices into their daily teaching, and in turn help their students learn math topics not typically taught in preschool.

These lessons and the general study protocol were piloted in three preschool classrooms. The pilot assessed children's knowledge on the topic they would learn about during a pre-test (Visit 1), videotaped the teacher teaching a lesson to a small group of children during Visits 2, 3, and 4, and assessed children's knowledge again during a post-test (Visit 5). Researchers provided two of the lessons the teachers taught. The teachers developed the third lesson as a follow-up to the two that were provided. After each lesson in Visits 2, 3 and 4, we videotaped feedback from the teachers about what they noticed, liked, and might use again from the lessons.

The pilot process revealed that the lessons were an appropriate length and at an appropriate level of difficulty for preschool children. The teachers found the lessons to be teacher-friendly and enjoyed both the content and the methods employed. They noted that the lessons were on topics that they had not considered teaching to their students, and they were surprised by how capable their children were of reaching new academic heights. One teacher noted that she had never thought to have children describe and discuss shapes for which they did not have names, and that she was surprised at how detailed and rich their descriptions were. Teachers also noted that children were actively engaged throughout each lesson. Teacher feedback will be used to revise and improve the lessons and protocol. The revised lessons and protocol will be piloted with a much larger group of teachers, who will teach strategy-enriched lessons and non-strategy-enriched lessons (on different topics), to allow us to compare their teaching strategies in each case. In order to measure the effect of the lessons on children's learning, pre and post-tests will be administered to students who receive the enhanced and unenhanced lessons as well as to children who do not experience the lessons at all.

Building children's early numerical understanding

In the area of early numerical understanding, we have developed and tested a specific intervention designed to target the numerical skills of cardinality and the successor function. The intervention takes the form of a game where children give puppets the number of objects the puppets like to have, and, along with the experimenter, count and label the objects and discuss the relations between different numbers of objects. As part of the design process, expert preschool teachers met and discussed the ideas for the intervention. Each teacher tried out the intervention with a preschool-aged child. Teachers then met as a group to talk about their impressions of the task and how to improve it. Based on teacher feedback, the intervention was modified to be more interactive and more cognitively engaging for the child.

The next step involved recruiting preschool classrooms to participate in a pilot study to test the intervention task with a larger number of children. The results from the pilot were very promising. Of those children who had not yet learned the cardinal principle, half of the children improved from pre-test to post-test on their cardinal number knowledge, resulting in an overall improvement in children's knowledge. Based on these promising pilot results, we designed an experimental study comparing children in the intervention condition to a control condition that involves only counting, but does not involve labeling the number of objects or discussing the relations between numbers. We chose to use counting as our control condition because this is the type of interaction that typically occurs in preschool classrooms, and thus it allowed us to determine whether our intervention results in more learning than "business-as-usual." This study involves children in five large, primarily low SES preschools. We are currently collecting data, randomly assigning children within classrooms to the intervention or control condition. We measure children's understanding of the successor function using an ordering task, where children are asked to put cards in order, where the cards represent the numbers 1-7 using Arabic numerals, dots, and hand shapes. For children who have already learned the cardinal principle (CP-knowers), we are finding that both our experimental and control conditions result in significant learning of the successor function, controlling for pre-test ordering accuracy and age.

While our analyses of this finding are ongoing (i.e., we are comparing both conditions to a non-numerical input control condition), the findings suggest that the counting-only condition, which was meant to be a business-as-usual control, may have actually represented substantially more numerical input than children typically receive in their preschool classrooms. Although CP-knowers improved after training, we did not see this effect for 3- or 4-knowers, children who understand the cardinal meanings of 3 or 4 number words but have not yet learned the cardinal principle – that is, they have not learned that for all of the numbers in their count list, the last number reached in counting a set represents the set's cardinal value. This suggests that children are ready to learn the successor function only after they have learned the cardinal principle. These findings have important implications about the sequencing of instruction for foundational number concepts.

In addition, we have found that children who had not yet learned the cardinal principle (also referred to as "subset-knowers" since they know only a subset of the number words in their count list), showed greater cardinal number knowledge in gesture than in speech. This was especially true of children who knew the cardinal meanings of fewer than 3 number words. We gave children two tasks where we showed them a picture of a certain number of objects (e.g., 3 boats) asked them "What's on this Card?" In one task, children were asked to show the answer with their fingers (WOC-Manual), and in the other, with their words (WOC-Verbal). Our results show that children who had not yet learned the cardinal principle were significantly more accurate on the WOC-Manual task than on the WOC-Verbal task for sets of size 2 and 3. In other words, the same child who, when shown a picture of 3 boats, would say an incorrect number like "five" was able to correctly show the number of boats using three fingers.

When they were shown larger sets of objects (5-10), subset-knowers gave responses that were closer to the correct number on the WOC-Manual than on the WOC-Verbal task, indicating that they were better able to approximate the number of objects in gesture than in speech.

We have also examined the occurrence of simultaneous gesture-speech mismatches on the WOC tasks. Gesture-speech mismatches occur when a child raises a certain number of fingers and, at the same time, says a different number word. We found that mismatches were more likely in response to set sizes for which children had not already learned the cardinal word meaning, and that children's gesture responses were more accurate than their verbal responses during these mismatches.

Additionally, we have investigated children's ability to estimate how many objects they see without counting (e.g., to see 9 dots on a screen and say "ten"). Before learning the cardinal principle, some children were able to give approximate responses by saying larger number words for larger set sizes (up to 10). This ability was related to their age but *not* to their ability to label a number of objects exactly. This suggests that numerical estimation begins to develop in preschool and appears to follow a somewhat separate trajectory than exact cardinal-number knowledge.

Another issue we explored is whether it is more effective for children to engage with multiple examples of small numbers (e.g., repeated examples of 1, 2, and 3) or to engage with examples of large (4, 5, 6) numbers, where set size cannot be perceptually ascertained but must be assessed by counting. We are currently testing this question through an intervention study in which parents are given either small number (1-3) books, large number (4-6) books, or books that discuss non-numerical properties (descriptive adjectives, e.g., the furry dog). Preliminary results suggest that children who read books about number (either small or large numbers) develop greater number knowledge over the course of a month than children who read books about other properties. Moreover, their numerical knowledge moves much more rapidly than would be expected based on reports in the literature. Currently, both types of number books appear to be equally effective in supporting numerical development. After data collection is complete, our analyses will test whether the relative effectiveness of each type of number book is determined by children's number knowledge at the start of the study (prior to receiving the intervention).

Building children's early geometry and spatial thinking

In the domain of geometry/spatial thinking, our efforts are aimed at going beyond typical early mathematics curricula, which focus almost entirely on teaching children to name 2-dimensional shapes. We are collecting data for a study where we use analogy, widely established in the cognitive and developmental psychology literatures as a powerful learning tool, to help children learn the defining attributes of the shape category "triangle". We have also collected data for a study that uses language to help very young children to understand the concept of "angle"—a concept that is not introduced into typical curricula until second grade. We are also working on developing instructional strategies that enhance children's spatial visualization skills—skills that are important but are not typically covered by early math curricula. Since feedback from teachers indicates that they find game play and gesture to be easy-to-implement learning tools, our interventions focus on the possible relative benefits of both iPad technology and gesture in improving spatial visualization skills.

<u>Improving children's shape-related concepts</u>. Young children's representations of geometric shape categories (e.g., *triangle*) tend to be centered on familiar, prototypical exemplars of shape categories. According to one study (Satlow & Newcombe, 1998), children typically have a prototype (e.g., an equilateral triangle with a horizontal side at the bottom), and when determining whether other exemplars are a member of the category, they simply use visual similarity to the prototype. This leads to rejection of atypical exemplars, e.g., scalene triangles or triangles with the horizontal side at the top, as members, and acceptance of non-members, e.g., a three sided but not enclosed shape, as members—these misconceptions persist late into development (up to age 9 for many children). We have created a bank of many different looking triangles (and almost-triangles) and have presented it to over 200 3.5 – 5 year old children. Our results mimic the ones suggested by the literature—at pretest, children

are likely to reject strange-looking triangles but happily accept non-triangles that are visually similar to the prototype.

However, shape category membership is determined by defining attributes (e.g., a triangle is an enclosed shape with three straight sides). We developed an intervention that uses comparing typical and atypical exemplars, and contrasting exemplars with non-exemplars to foster learning the attributes of the shape category “triangle”, and have collected data with over 200 children. We find that practicing comparison or contrast of any kind improved children’s triangle category knowledge. Beyond that, we found that comparing two examples, (e.g., “This is a triangle and this is a triangle”) helped children to accept more strange-looking triangles as members of the triangle shape category. Contrastive examples (e.g., “This is a triangle, this is not a triangle”) helped children to sharpen their shape category—they were more likely to reject non-exemplars. Further, we found that contrasting two forms, i.e., a scalene triangle (“this is a triangle”) and an identical form that is missing part of one side (“this is not a triangle”), helps children to learn the rule “triangles have enclosed sides”. This type of contrast is also effective at teaching children the rule that triangles must have *exactly three* of those enclosed sides. Notably, the effectiveness of contrastive examples ran counter to teachers’ intuitions that comparison would be most effective in teaching children the triangle concept. We are currently analyzing these data and will use the findings to create pilot lessons.

<u>Improving children’s understanding of “angle”.</u> We have investigated the role of language in children’s misconceptions regarding the early geometric concept of angle. When making judgments about the size of angles, children often incorrectly rely on erroneous properties such as the overall size of the angle-figure (Lindquist & Kouba, 1989; Clements & Battista, 1990). One possibility is that this misconception stems from the whole object bias -- the tendency to assume that a novel label refers to an object as a whole rather than to its parts or features (Markman, 1994). By this account, when presented with a canonical angle figure (two line segments meeting at a point), children may believe “angle” refers to the entire figure and therefore think that a “bigger angle” is a larger figure and not a figure with a larger measure of rotation. Given another constraint on word learning, mutual exclusivity, we hypothesized that children would be more likely to attach the correct referent to the word “angle” if they were first given a label for the overall angle shape (e.g., “toma”).

To test this, we carried out a study of preschool children using a single session pretest/training/post-test design. During piloting and the study itself we ran a total of 61 participants, although only 30 of these participants completed the final version of the study (mean age: 4.86 years, SD: 0.53 years). At pretest, children showed evidence of the whole-object misconception. After training, children who were given a novel-word label for the whole object improved significantly more than those trained on the meaning of “angle” alone. These results suggest students' angle misconceptions are not due to perceptual or representational limits, but rather they are due to the misleading language typically used to describe angles. This opens the possibility that the concept of angle could be introduced far earlier in school as long as these barriers are addressed.

Improving spatial visualization and spatial language. Young children's dynamic spatial visualization skills such as mental rotation are related to later spatial and math abilities, predict academic performance and entry in the STEM (Science, Technology, Engineering, and Mathematics) disciplines over the lifespan, and importantly, can be improved. We are assessing the effectiveness of a variety of instructional strategies that use technology to improve young children's mental rotation abilities. We have so far completed two pilot studies, one investigating the relative benefits of action vs. gesture on tangible manipulatives as a way to improve mental rotation ability, and one investigating the relative benefits of actually moving objects vs. gesturing the movement of the objects as a way to improve mental rotation ability. The findings of these studies indicate that both gesture and action are effective in improving spatial visualization, but that their effects unfold over different timetables – action more quickly and gesture more slowly. These findings raise the possibility that gesture training may be more durable over time. We are also interested in whether the effects of gesture training may be more generalizable. We believe this may be the case because action training is tied to the objects being manipulated and the results of the action are visible whereas gesture encourages children to visualize the results of the rotation.

Our pilot work has led to a five-condition study where we try to untangle the mechanisms behind possible relative benefits of spatially dynamic actions and gestures. Four-year-old children receive instruction in mental rotation with a computer game. Following a pretest, children practice doing more mental rotation problems, along with some kind of action. In the Object condition, children rotate actual cut-outs of animals on a foam-board. The *action* in the 2-finger iPad condition is identical, except that the animals are displayed on an iPad screen. The Gesture condition has the same action, except the animals on the iPad do not move in response to the child's action. The action in the 1-finger iPad condition is different in that children do not "grasp" the animals but rather turn them by touching with one finger and tracing a circle (note this is similar to many interactions with virtual manipulatives. Finally, in the Click condition, children click an animal that rotates on its own. We have begun data collection on this study (following an extensive piloting phase), and have collected data from about 40 children so far. Preliminary results show that all of the action conditions as well as the gesture conditions lead to better improvement on MR than does the Click condition (our control condition). These promising preliminary results support our hypothesis that rotation actions and gesture will help children improve their MR abilities. We are interested in whether the action and gesture conditions might differ depending on the level of children's skill, a question we are pursuing as we continue to collect data.

Teacher Work Circle

In the summer of 2012, fourteen Pre-Kindergarten and Kindergarten teachers joined researchers for a two-day long work circle. The work circle started with overviews of early math followed by a discussion of instructional dilemmas teachers face. Teachers reported needing help in deciding which topics to teach during math, given a lack of common core standards and curricula in pre-K. During each day, professors, post-doctoral researchers, graduate students, and research assistants gave informal presentations of lab research in topics like numeracy, measurement and estimation, angle learning, math anxiety, and spatial visualization. Each presentation was followed by a 45-minute breakout session, where teachers and researchers brainstormed instructional dilemmas and strategies for solving them, based on the empirical research presented. At the end of each day, teachers worked in groups to come up with instructional activities for the classroom. At the end of the final day, teachers shared their instructional activities with one another. Teachers then received a file with notes from the instructional activities they had created.

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Related Papers

1. Gibson, D.J., Congdon, E.L., & Levine, S.C. (in press). Effects of word-learning biases on children's concept of angle. *Child Development*.

2. Gunderson, E.A., Spaepen, E., & Levine, S.C. (revise/resubmit). Approximate number word knowledge before the cardinal principle.

3. Gunderson, E. A, Spaepen, E., Gibson, D., Goldin-Meadow, S., & Levine, S.C. (under review). Gesture as a window onto children's number knowledge.

4. "Research and Practice in the Field of Early Mathematics Learning." (November 2013). Getting on Track Early for School Success: www.norc.org/gettingontrack

5. "Getting on Track Early for School Success: Project Overview." (November 2013). Getting on Track Early for School Success: www.norc.org/gettingontrack

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